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RESEARCH ARTICLE - ENGINEERING

Finite Element Modeling of Saint-Venant Equations for Shatt-Al Hilla

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Abstract

Shatt Al-Hilla was considered one of the important branches of Euphrates River that supplies irrigation water to millions of dunams of planted areas. It is important to control the velocity and water level along the river to maintain the required level for easily diverting water to the branches located along the river. So, in this research, a numerical model was developed to simulate the gradually varied unsteady flow in Shatt AL-Hilla. The present study aims to solve the continuity and momentum (Saint-Venant) equations numerically to predict the hydraulic characteristics in the river using Galerkin finite element method. A computer program was designed and built using the programming language FORTRAN-77. Fifty kilometers was considered starting from downstream of Hindiyah Barrage towards Hilla city. The gathered field measurements along different periods were used for the purpose of calibration and verification of the model. The results show that the suitable Manning roughness was 0.023. A comparison with field observations was conducted to identify the validity of the numerical solution of the flow equations. The obtained results indicate the feasibility of the numerical techniques using a weighting factor of 0.667 and a time increment of 6 hr. High accuracy and good agreement were achieved, and minimum Root Mean Square Error (RMSE) of 0.029 was gained for the obtained results compared with the corresponding field observations.

Keywords: River simulation, Numerical modeling, Galerkin finite element.

1. Introduction

The study of water resources projects and hydraulic constructions, their development, and management has become one of the major concerns of society. The escalating demands of limited water resources and the need for maintaining water quality suitable for human, agricultural, and industrial uses and the complex interactions of the numerous elements of the man-water environments have necessitated the use of more sophisticated forecasting, designing, and management of water resources systems [1]. Shatt Al-Hilla was considered one of the important main branches of Euphrates River and irrigates large scales of planted areas in the middle of Iraq. The selected reach of Shatt Al-Hilla is extended in alluvial soils, and pass through successive planted areas located along, and directly surpass the water of the river [2]. So, it is necessary to study the flow characteristics in the river to specify the flow issues and to control the velocity and water level along the profile to maintain the required water level and to easily deliver the water to the branches located along the reach, as well as to avoid the sedimentation problems [3]. Approximately fifty kilometers length of Shatt Al-Hilla was considered in the present research. Fig. (1) illustrates the layout of the reach of the study. This work was supported by field measurements for a period of thirty days for the purpose of field application of the model. Flow in streams is seldom steady, the mathematical model derived from the continuity and the momentum, the Saint-Venant equations, can simulate the gradually varied unsteady flow in a river reach. These are generally expressed as one-dimensional nonlinear partial differential equations [4,5]. There is no analytical solution for these equations, so numerous finite difference methods have been developed for solving these equations using different schemes and approaches [6,7]. The finite element method in one form or another exists as a numerical procedure for solving these equations and others, but it did not gain enough attention until powerful computers become available [8,9]. The Weighted- Implicit Galerkin finite element method was adopted herein for solving the Saint-Venant equations [3,10]. The Galerkin method is a particular weighted residual finite element method in which the weighting function (N^T) is the same as the shape function (*N*) [9]. The general objective of the present study is to solve the Saint-Venant Equations to simulate the flow characteristics of the unsteady-state flow in Shatt Al-Hilla using numerical techniques, namely Galerkin finite element method. As well as simulating the unsteady flow in Shatt Al-Hilla to be considered as a case study for testing the validation of the numerical solution.

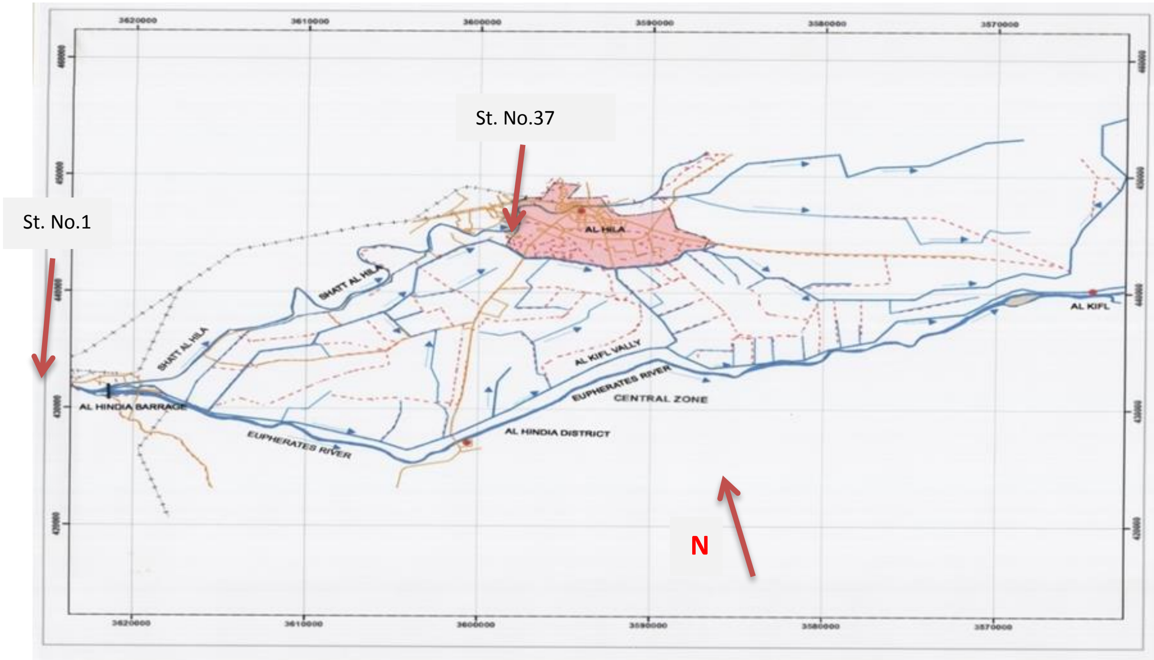

Fig. 1. Layout of the study reach of Shatt Al-Hilla (Authority of Hill-Kifel Project).

2. Theoretical Concepts

The basic flow equations mathematically describing unsteady river flow are the continuity equation, derived from the principle of conservation of mass, and momentum equation, derived from the principle of conservation of momentum. The derivation details of Saint-Venant equations may be found in standard works of hydraulics [2, 5].

The mass and momentum conservation equations (the hydrodynamic part of the model) can be written as;

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} \mp q = 0 \qquad (1)$$

and,

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left({Q^2}/{A}\right) + gA\left(S_f + S_0 + \frac{\partial y}{\partial x}\right) \pm q\,\frac{Q}{A} = 0 \qquad (2)$$

where,

Q= Discharge (m^3/sec)

A=Cross-sectional area (m^2)

q = Lateral inflow ($m^3/sec/m$)

y = Water depth (m)

S_0= Bed slope

S_f =Energy line slope obtained using Manning's equation

$$S_f = {Q^2.n}\Big/\left(A^2\,R^{\frac{4}{3}}\right)$$

n = Manning's roughness coefficient

R = Hydraulic radius (m)

X = Distance (m)

t = Time (sec)

The positive and negative signs in last terms of Eqs. (1) and (2) refers to lateral inflow and outflow, respectively.

3. Numerical Solution

The reach of the river was divided into 36 elements (37 sections or nodes). For the purpose of keeping the calculations simple, the linear shape function was selected, and the time cost will be minimal. If a function defined as $y(x, t)$ (or y for simplicity) was assumed to be linearly varied in the elements forming the flow field, the following relationship results [2, 5]:

$$y = N_1^e\, y_1 \; + \; N_2^e\, y_2 \tag{3}$$

In which $N_1^e = 1 - {}^{x}\!/_{L}$, and $N_2^e = {}^{x}\!/_{L}$ are the linear shape function of the elements at nods 1 and 2, respectively. Similarly, it can also be written for other variables.

Application of the Galerkin weighted residual principles to Eqs. (1) and (2) will produce a system of non-linear ordinary differential equations with respect to time. The mathematical expressions for these equations are:

$$\sum_{1}^{k-1} \int_0^L N^T \left[\frac{\partial Q}{\partial X} + \frac{\partial A}{\partial t} - q_L\right] dx = 0 \tag{4}$$

and

$$\sum_{1}^{k-1} \int_0^L N^T \left[\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left({}^{Q^2}\!/_{A}\right) gA\left(S_f - S_0 + \frac{\partial y}{\partial x}\right) + q_L \frac{Q}{A}\right] dx = 0 \tag{5}$$

In which summation of formulation equations was carried out for individual element equations, from element (*k-1*); and N^T represents the transpose of the shape function *N*.

Evaluation of the individual terms presented in Eqs. (4) and (5) which extends over the length of each element will yield the following non-linear ordinary differential form of finite element equations;

$$\frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} A_1^{\bullet} \\ A_2^{\bullet} \end{Bmatrix} - \frac{q_L\, L}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 0 \tag{6}$$

and

$$\frac{L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} Q_1^{\bullet} \\ Q_2^{\bullet} \end{Bmatrix} + \frac{1}{6}\begin{bmatrix} (Q_2 - 4Q_1) & (2Q_2 - Q_1) \\ -(Q_2 - 2Q_1) & (4Q_2 - Q_1) \end{bmatrix}\begin{Bmatrix} \left({}^{Q}\!/_{A}\right)1 \\ \left({}^{Q}\!/_{A}\right)2 \end{Bmatrix} + \frac{q_L\, L}{12}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{Bmatrix} (As_f)1 \\ (As_f)2 \end{Bmatrix} - \frac{g\, S_0\, L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} A1 \\ A2 \end{Bmatrix} + \frac{g}{6}\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\begin{Bmatrix} (A_y)1 \\ (A_y)2 \end{Bmatrix}$$
$$+ \frac{q_L\, L}{12}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \left({}^{Q}\!/_{A}\right)1 \\ \left({}^{Q}\!/_{A}\right)2 \end{Bmatrix} = 0 \tag{7}$$

where, dots, •, refer to time differentiation.

For conducting the formulation of Eqs. (6) and (7) in a finite element form as a grid which consists of multi-elements, they will be arranged in a matrix form contains the summation of *N* elements. This summation form usually needs to call the elements assembly.

The form of a global assembled matrix equation for Eqs. (6) and (7) will be;

$$[B\,]\{Q\} + [C]\{A^{\cdot}\} - \{D\} = 0 \tag{8}$$

and

$$[E]\{Q^{\cdot}\} + [F]\{Q/A\} - [G]\{A\} + [H]\{A_y\} + [I]\{AS_f\} = 0 \tag{9}$$

where the brackets [ ] and { }refer to the global matrix and the vector of the global column, respectively.

When the dimensionless time-weighting factor $\emptyset$ is applied as well as the forward difference scheme is implemented to Eqs. (8) and (9), which will result in the following non-linear algebraic Eqs. (10) and (11):

$$\left[\emptyset\Delta tB\right]\{Q\}^{j+1}+[C]\{A\}^{j+1}-\emptyset\Delta t\{D\}^{j+1}=\left[(1-\emptyset)\emptyset\Delta tB\right]\{Q\}^{j}+[C]\{A\}^{j}-(1-\emptyset)\Delta t\{C\}^{j} \tag{10}$$

$$\begin{aligned}[E]\{Q\}^{j+1}-\emptyset\Delta t[F]\{Q/A\}^{j+1}-\emptyset\Delta t[C]\{A\}^{j+1}-\emptyset\Delta t[H]\left\{A_y\right\}^{j+1}+\emptyset\Delta t[I]\left\{AS_f\right\}^{j+1}\\ =[E]\{Q\}^{j}+(1-\emptyset)\Delta t[F]\{Q/A\}^{j}-(1-\emptyset)\Delta t[G]\{A\}^{j+1^{j}}-\emptyset\Delta t[H]\left\{A_y\right\}^{j}\\ +(1-\emptyset)\Delta t[I]\left\{AS_f\right\}\end{aligned} \tag{11}$$

where, $\emptyset$ can vary between 0 and 1.

The generalized iterative functional method that is well-known as the Newton- Raphson method [4] is employed for the simultaneous solution of Eqs. (10) and (11) which can be written in a functional form as:

$$\left.\begin{aligned}
&\left.C_1(y_1,Q_1,y_2,Q_2)=0\right\} \quad upstream\ node\\
&M_1(y_1,Q_1,y_2,Q_2)=0\\
&\left.\begin{aligned}&C_i(y_{i-1},Q_{i-1},y_i,Q_i,y_{i+1},Q_{i+1})=0\\ &M_i(y_{i-1},Q_{i-1},y_i,Q_i,y_{i+1},Q_{i+1})=0\end{aligned}\right\} \quad mid\ node\\
&\left.\begin{aligned}&C_1(y_1,Q_1,y_2,Q_2)=0\\ &M_1(y_1,Q_1,y_2,Q_2)=0\end{aligned}\right\} \quad downstream\ node
\end{aligned}\right\} \tag{12}$$

The number of elements between upstream and downstream boundaries is (*N-1*). Thus total equations of (*2N-2*) with (*2N*) unknowns are obtained. The upstream boundaries provide the additional two equations required to complete the needed number of equations for the system. The upstream boundary condition consists of a rating curve measured at station No. 1 at almost one-hour intervals, Fig. (2). While the downstream boundary condition consists of a stage hydrography measured at station No. 37 for the same interval, Fig. (3). A computer program was designed and built as a part of the present research and the programming language FORTRAN-77 was employed to write the program.

4. Results and Analysis

For the purpose of testing the numerical model, field observation along a time period of eleven days is prepared and the data is roughly separated into two groups. The first group is used for the calibration of the model, while the second group is used for verification. The calibration of Shatt AL-Hilla simulation mainly involves the flow resistance representation by Manning's roughness coefficient (n), and the choice of time step Δt and weighting factor $\emptyset$, to ensure convergence and accuracy of the solution.

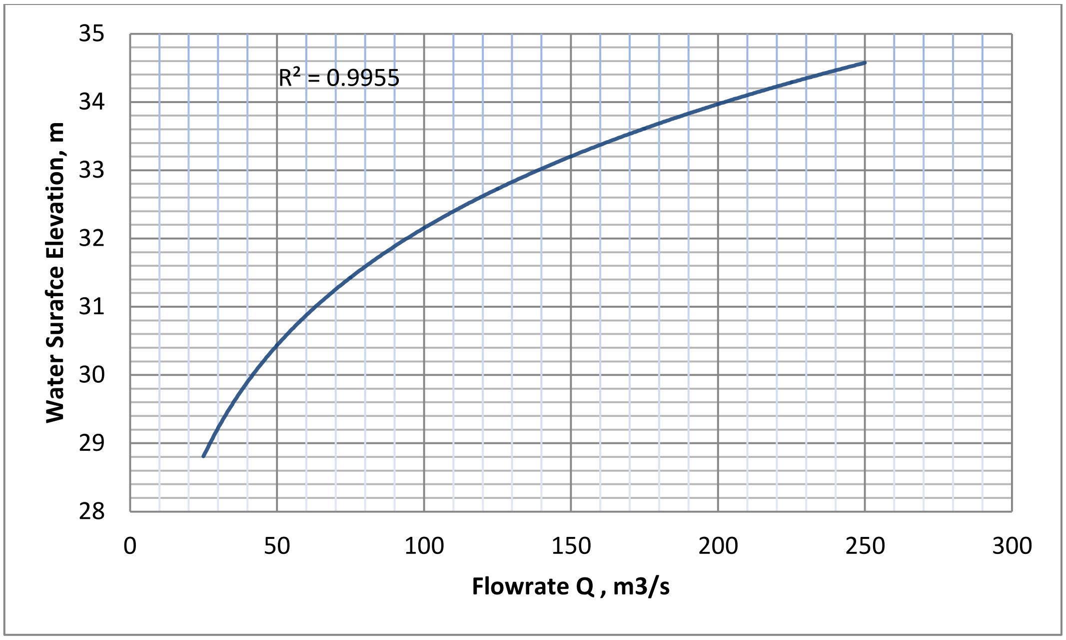

Fig. 2. Upstream boundary condition: Rating curve of (Shatt Al-Hilla) at station No.1.

A series of computations were performed with specified values of $\emptyset = 0.667$, $\Delta t = 6$ hours, and lateral inflow $q_L = 10\ m^3/sec$. These computations were conducted with different Manning coefficient ranging between 0.10 and 0.04, Fig. (4). The results indicate that the appropriate value of Manning coefficient was 0.023 for the studied reach of the river, which achieves good agreement with the observed water level, due to the smallest value of RMSE of 0.016.

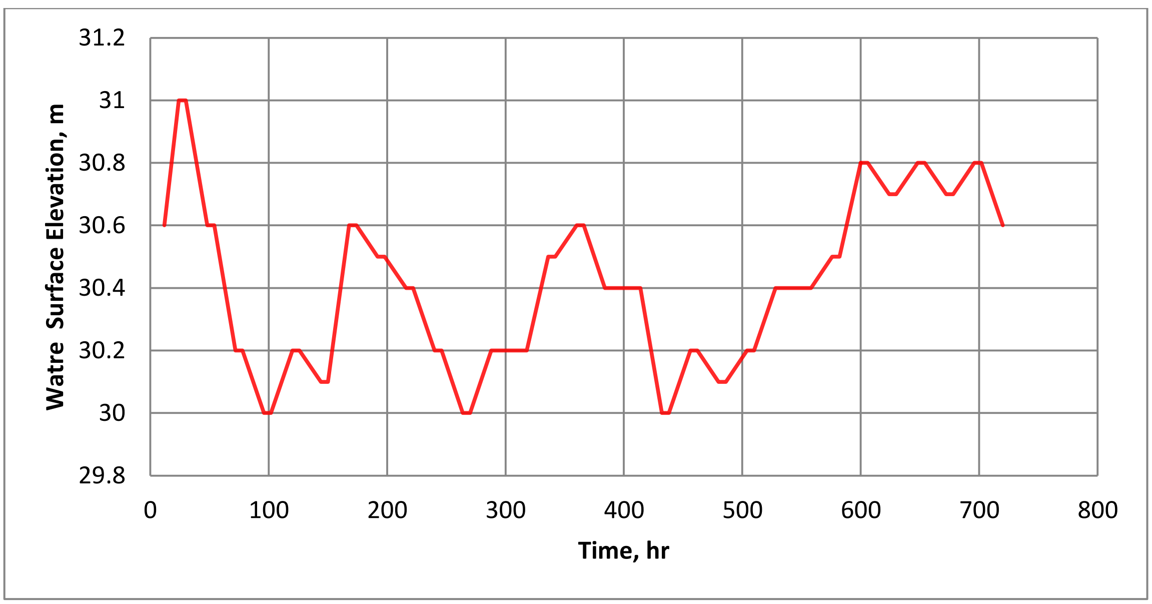

Fig. 3. Downstream boundary condition: Stage hydrograph of (Shatt-Al Hilla) at station No.37.

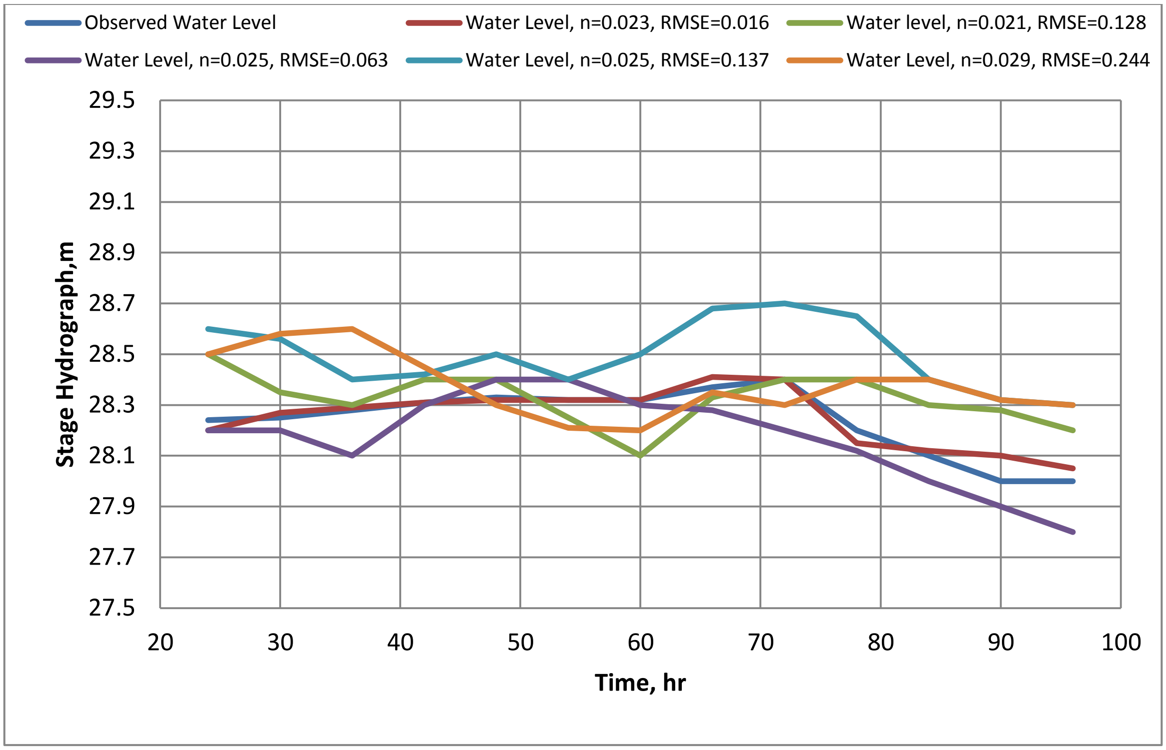

Fig. 4. Effect of Manning Roughness, (*n*), on the estimated stage hydrograph at Shatt-Al Hilla (Δt =6 hr and $\emptyset$ =0.667).

Another series of computations were conducted for the choice of convenient values of the parameters that control the numerical computations, which have an effect on the accuracy and convergence. These parameters are $\emptyset$ and Δt.

Figs. (5) and (6) show the effect of different values of $\emptyset$ and Δt on the results obtained by the simulation model, respectively. The comparison illustrated in these figures between the calculated and observed values indicates that the appropriate value $\emptyset$ is 0.667. The time increment $\Delta t =$ 6hr will provide quick convergence and accurate results. These values of weighting factor and time increment produced smaller values of root mean square error, RMSE, of 0.036 and 0.034, respectively. When these parameters' values were used, acceptable accuracy results were obtained, and minimal time was costed. It is important to mention that an appropriate tolerance limit was specified for the terminations of iteration in a minimum time period.

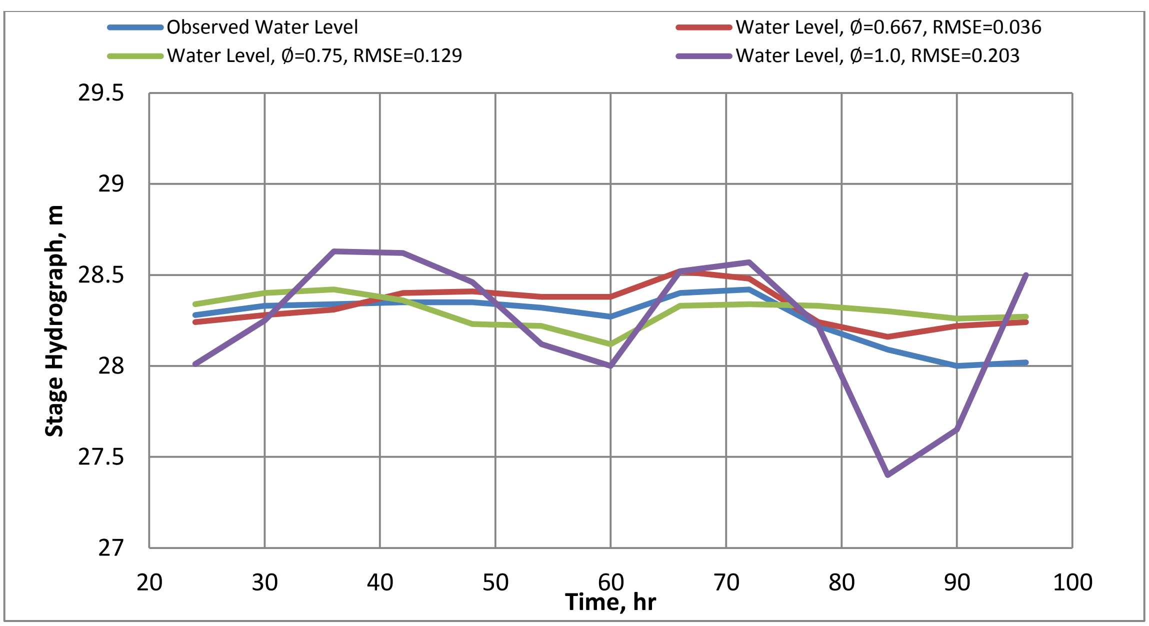

Fig. 5. Effect of weighting factor, $\emptyset$, on the estimated stage hydrograph of Shatt-Al Hilla (*n*=0.023, Δt=6 hr).

Verification of the model was achieved using the second group of gathered data with Manning coefficient n=0.03, $\emptyset = 0.667$, lateral inflow q_L =10 m^3/sec, and Δt = 6 hours. Fig.(7) illustrates the validation of the numerical model and the good agreement of the results obtained from the simulation model with the observed stage values. The achieved RMSE was 0.029.

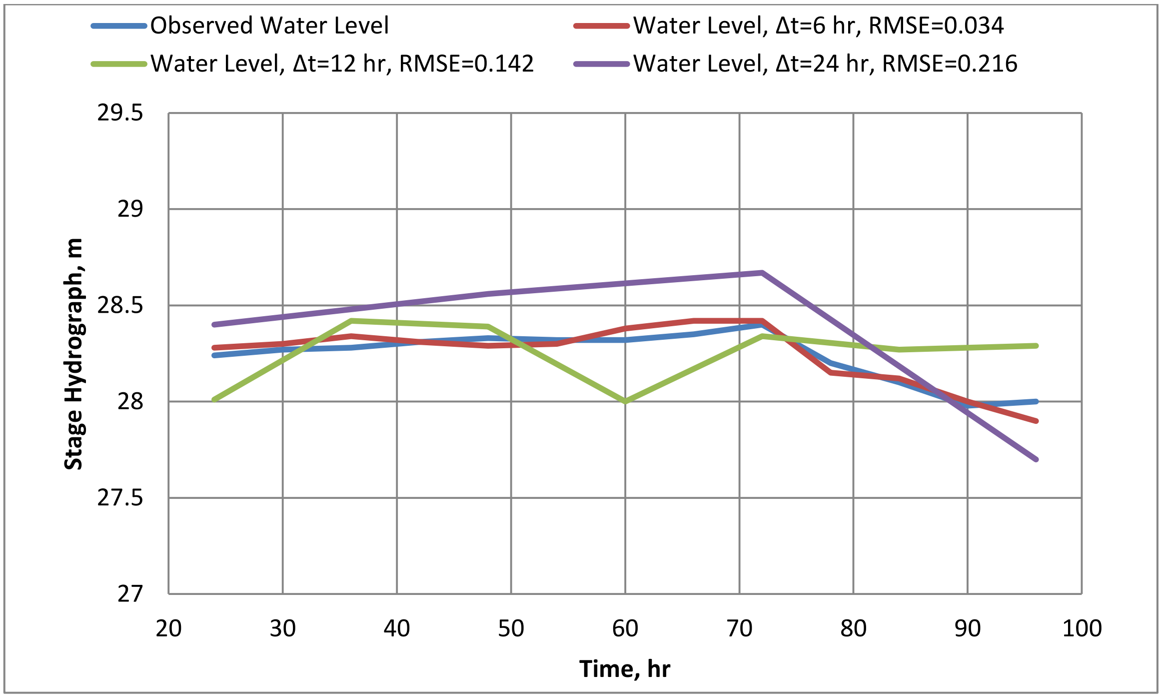

Fig. 6. Effect of time interval, Δt, on the estimated stage hydrograph of (Shatt Al-Hilla) (n=0.023, Ø=0.667).

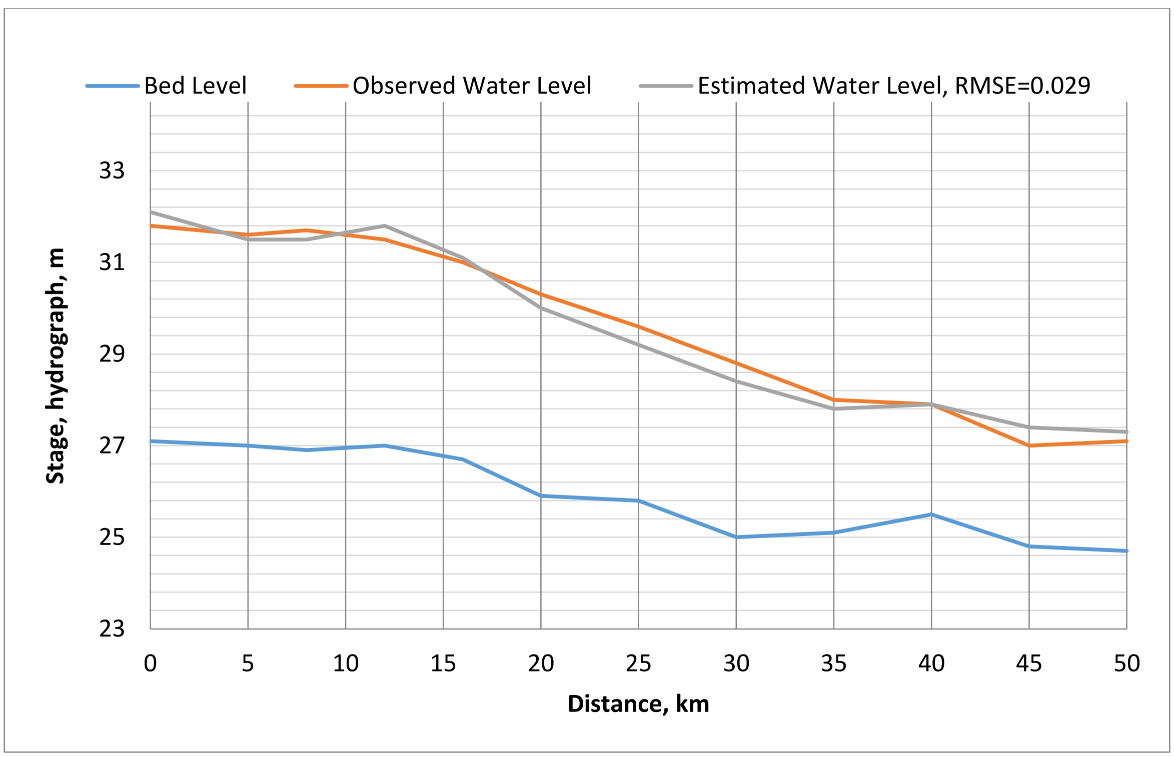

Fig. 7. Estimated and observed longitudinal water surface profile of the reach of (Shatt Al-Hilla) (n=0.023, Δt=6 hr, Ø=0.667).

5. Conclusions

In this research, the effect of the Manning roughness, time weight-factor, and time interval were studied to investigate their effects on the solution of the Saint-Venant equation using the finite element method. As a result, there are many outcomes of the present study which can be presented as;

1) A weighted-implicit Galerkin finite element method produces stable and an unconditional solution of unsteady flow (Saint-Venant) equations with high accuracy.

2) High agreement with observed values, with an acceptable RMSE value, was obtained for the yielded stage hydrograph and rating curve when the numerical model was applied to Shatt AL-Hilla as a field problem with specified flow conditions.

3) The appropriate value of Manning coefficient (n) appears to be 0.023 for the reach of the river considered in the present study.

4) The appropriate value of the computational parameters $\emptyset$ and Δt are 0.667 and 6 hr, respectively.

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