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RESEARCH ARTICLE

Shell Model Calculations of Energy Levels and Binding Energy of $^{133;135}$Sn, and $^{133;135}$Sb Nuclei

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Abstract

Unrestricted calculations of the shell model were conducted to investigate the positive and negative energy levels and binding energy for A-odd nuclei $^{133;135}$Sn and $^{133;135}$Sb by utilizing the effective interactions kh5082, jj56pna, khhe, and jj56cdb. The predicted results have been compared with the recently measured data and good global agreements were obtained for all interactions except the khhe effective interaction failed to determine the ground state of odd nuclei, it is concluded that there is no universally effective interaction in this mass region and more investigation needed to study other nuclear structure properties that might help us to better understand the nuclei lies in the vicinity of ^{132}Sn core.

Keywords: Shell Model; Effective Interactions; ^{132}Sn region.

1. Introduction

The nuclei lying around the doubly-magic ^{132}Sn region is a major topic in both theory and experiment right now. New enhanced techniques and facilities, such as those that employ radioactive ion beams, enable the collection of new data [1–3], allowing theoretical models to be tested. In particular, microscopic methods should be used to investigate or seek for possible development of shell structure as the neutron drip line approaches [4,5]. Indeed, the so-called "N=82 shell-quenching" has been suggested to play a role in the creation of the A=130 peak abundance of the r-process in a recent paper [6]. The two-body matrix elements (TBME) of the shell-model effective interaction receive special emphasis in this context. C. Radford et al. [7] used the excitation of Coulomb for radioactive beams to measure the values of B (E2) for the initial 2+ excited states of neutron-rich ^{132,134,136}Te. The values of B(E2) found for ^{132,134}Te are in great accord with the systematics of heavy stable Te isotopes, but the value for ^{136}Te is unexpectedly small. The Holifield Radioactive Ion Beam Facility (HRIBF) has developed neutron-rich radioactive beams, which has stimulated experimental and theoretical research in heavy Sn and Te isotopes. B. A. Brown et al. [8] reported a new calculated hell model shell model for magnetic moments for selected $^{130-132}$Sn and $^{132-134}$Te isotopes. Both realistic CWG [9-12] and empirical SMPN (1+2)-body Hamiltonians have been used in large-basis unconstrained shell-model (SM) computations for Sn [10] the two theoretical outcomes are vastly different. The CWG realistic interaction predicts the 2_1^+ for the even-even Sn isotopes above the ^{132}Sn core. The realistic interaction CWG predicts almost constant energies of 2+1 states, which are generally predicted for singly magical nuclei.

The purpose of this paper is to apply the unrestricted large-scale shell model calculation to study the positive and negative energy levels including high J^p values and binding energy for $^{133;135}$Sn, $^{133;135}$Sb isotopes near the ^{132}Sn doubly-magic core. The calculation will be performed for selected isotopes using the nuclear shell model code NushellX@MSU utilizing the model space j j56pn, by employing the effective interactions j j56pna, j j56pnb, kh5082, cw5082, jj56cdb, and khhe. The results of the calculations of the level schemes' reduced transition probabilities and binding energy will be compared with the recently available measured data.

Nomenclature & Symbols			
Sn	Tin is a chemical element with the symbol Sn (from Latin: Stannum)	TBME	Two-Body Matrix Elements
HRIBF	Holifield Radioactive Ion Beam Facility	SM	shell-model
Sb	Antimony is a chemical element with the symbol Sb (from Latin: Stibium)	J^P	Total angular momentum with parity (P)
CWG	Chung-Wildenthal G Matrix	SMPN	Shell Model Proton Neutron

2. Shell Model Calculations

The calculations were conducted in the jj56pn model space for isotopes $^{133;135}$Sn and $^{133;135}$Sb near the doubly magic core ^{132}Sn by using the windows-based recent code NushellX@MSU without imposing any restriction to the model space by utilizing the effective interactions jj*56pna*, *kh5082*, *jj56cdb*, and *khhe*. The calculations of excited level spectra and the binding energy from the present study were compared with the recent available experimental data and a conclusion had been drawn from the present study.
In the nuclear shell model, the center mean-field potential is created by the individual nucleons.
In the case of many-body issue, such as in nuclear physics. To write the Hamiltonian as the sum of the components for the individual particles in the nucleus, start with the precise of independent particle motion [11],

$$H^{(0)} = \sum_{i=1}^{A}[T_i + \mathrm{U\ (ri)}] \tag{1}$$

In which H(0) denotes the Hamiltonian unperturbed system and U (r_i) and T_i are the potential and kinetic energies, respectively. The Slater-type eigenfunctions are the solution to eq.(1), that obeys the principle of Pauli exclusion that wavefunctions for identical particles must be antisymmetric, therefore eigenfunctions for the two-particle system can be written as [11],

$$\phi_{\alpha 1,\alpha 2} = \frac{1}{\sqrt{2!}}\begin{vmatrix}\varphi_{\alpha 1}(1) & \varphi_{\alpha 1}(2)\\ \varphi_{\alpha 2}(1) & \varphi_{\alpha 2}(2)\end{vmatrix} = \frac{1}{\sqrt{2!}}[\varphi_{\alpha 1}(1)\,\varphi_{\alpha 2}(2) - \varphi_{\alpha 1}(2)\,\varphi_{\alpha 2}(1)] \tag{2}$$

where $\varphi_{\alpha 1}(1)$ is the product of radial and angular components for single-particle wavefunctions. Clebsch-Gordon coefficients, spherical harmonics, and the radial component are all included in this somewhat complicated equation. The radial portion of the expression, $R_{nl}(r)$, is the only part that isn't clearly specified (r). The shape of the eq's potential U(r) determines the features of eq. (1). It turns out that selecting this possibility isn't as simple as it appears. The Harmonic Oscillator (HO) is the most commonly used potential because the wavefunctions are integrable. The Woods-Saxon (WS) provides a more realistic potential but the problem cannot be solved only numerically [11],

$$U(r) = \frac{1}{2}m\omega^2 r^2 \quad \text{(Harmonic Oscillator)} \tag{3}$$

where m is the mass of the nucleon, ω is the oscillator's frequency, and r denotes the parameter of the radius [11],

$$\mathrm{U(r)} = \frac{U_0}{1+\exp\left(\frac{\mathrm{r}-\mathrm{R_0}}{\mathrm{a}}\right)}\ \text{(Woods - Saxon)} \tag{4}$$

where U_0 is the depth of the potential, a is the parameter of the diffuseness and $R_0=r_0A1/3$. The shell gaps found in nuclear data are not reproduced by inserting any of these equations into the Schrödinger equation. To adequately account for these "magic numbers," a substantial spin-orbit component must be introduced. The magnetic field generated by this apparent motion interacts with the nucleon's magnetic moment. The dot product $\vec{\ell}.\vec{S}$ of the two gives the energy of interaction between a magnetic moment and a magnetic field. The spin-orbit interaction energy must be proportional to the dot product [12],

$$U_{so} = f(r)\vec{\ell}.\vec{S} \tag{5}$$

Thus, the function $f(r)$ is connected to the central potential and contains the reliance on the radial coordinate r. The anticipated value f (r)n,l is predicted to be in the order of $\hbar\omega_0 \sim 41A^{-1/3}$ MeV based on experimental data [11,13]. The wavefunctions normalized and antisymmetrized for the system of "identical fermions" were obtained using the Slater determinant of eq. (2). Because the nucleus includes both neutrons and protons, the nucleus interacts with each other. In their current state, these wave functions can't be employed. Instead, an isospin new quantum number T is introduced to help distinguish between neutrons and protons. The core is then considered as an inert system, the valence nucleons, on the other hand, are dealt with independently [11],

$$\Phi_{J,\tau} \sim \Phi_{0,\tau}^{core} \times \Phi_{J,\tau}(\alpha_1, \alpha_2, \dots\dots) \tag{6}$$

The concept of independent-particle motion is impractical in this case, because the circling particles must undoubtedly interact with one another. As a result, an A-particle system's independent-particle Hamiltonian may be expressed in terms of two-particle interactions as [11],

$$H = \sum_{k=1}^{A} T_k + \sum_{k=\ell}^{A}\sum_{\ell=k+1}^{n} \mathrm{W}\,(\vec{\mathrm{r}}\mathrm{k}\,,\vec{\mathrm{r}}\ell\,), \tag{7}$$

where W ($\vec{\mathrm{r}}$k , $\vec{\mathrm{r}}\ell$) is the interaction between two-bodies for the k$^{\mathrm{th}}$ and ℓ^{th} nucleons. Taking an average potential U (r_k), the Hamiltonian reads,

$$H = \sum_{k=1}^{A}[T_k + \mathrm{U\ (rk)}] + \sum_{k=\ell}^{A}\sum_{\ell=k+1}^{n} \mathrm{W}\,(\vec{\mathrm{r}}\mathrm{k}\,,\vec{\mathrm{r}}\,\ell\,) - \sum_{\mathrm{k=1}}^{\mathrm{A}} U_{(rk)}\,, \tag{8}$$

The first term in the above equation is the same as the Hamiltonian for independent-particle provided by eq. (1), and the second and third terms account for the residual interaction, which is the divergence from independent particle motion. Equation (1) may be recast by separating the summations into core and valence contributions [11],

$$" H = H_{core} + H_1 + H_2 + \mathrm{V}\,(\vec{\mathrm{r}}_1\,, \vec{\mathrm{r}}_2\,)" \tag{9}$$

Here, H_{core} include all the interacting nucleons comprising the core, H_1 and H_2 are the contributions comes from the single particles 1 and 2, and "V $(\vec{\mathrm{r}}_1\,, \vec{\mathrm{r}}_2\,)$ " which include the interactions between particles 1 and 2, in addition to any interactions with core nucleons. The energy is expressed similarly when this form of the Hamiltonian is used in the Schrodinger equation [11],

$$E = E_{core} + E_1 + E_2 + \langle \Phi_{J,\tau} | \mathrm{V}(\vec{\mathrm{r}}_1\,, \vec{\mathrm{r}}_2\,) | \Phi_{J,\tau} \rangle \tag{10}$$

E_{core} core is the core binding energy, E_1 and E_2 are the energies of the single particle of orbitals of the core, and " $\langle \Phi_{J,\tau} | \mathrm{V}(\vec{\mathrm{r}}_1\,, \vec{\mathrm{r}}_2\,) | \Phi_{J,\tau} \rangle$" is the "residual interaction" in the preceding equation. Linear combinations of the unperturbed wavefunction generate the mixed eigenstates [11],

$$(\Psi_{J,\tau})p = \sum_{k=1}^{\mathrm{g}} a_{kp} \left(\Phi_{\mathrm{J}}, \tau\right)_k \ , \tag{11}$$

where the configurations number is g and the label p=1, 2, ..., g.

The a_{kp} coefficients should obey the condition,

$$\sum_{k=1}^{\mathrm{g}} |a_{kp}|^2 = 1 \tag{12}$$

Substituting eq. (11) into the equation of Schrodinger leads to,

$$H(\Psi_{J,\tau})p = E_p(\Psi_{J,\tau})p, \tag{13}$$

which gives the system's "linear equations" of the form [11]

$$\begin{pmatrix} \mathrm{H}_{11} & H_{12} & \cdots & H_{1\mathrm{g}} \\ H_{21} & H_{22} & \cdots & H_{2\mathrm{g}} \\ \vdots & \vdots & \ddots & \vdots \\ H_{\mathrm{g}1} & H_{\mathrm{g}2} & \cdots & H_{\mathrm{gg}} \end{pmatrix} \begin{pmatrix} a_{1p} \\ a_{2p} \\ \vdots \\ a_{\mathrm{g}p} \end{pmatrix} = E_p \begin{pmatrix} a_{1p} \\ a_{2p} \\ \vdots \\ a_{\mathrm{g}p} \end{pmatrix} \tag{14}$$

For particles l and k, the summing of all single-particle and residual interaction terms yields $H_{\ell,k}$. Equation (14) is a standard eigenvalue issue that is addressed by setting the determinant to zero [11] ,

$$\begin{vmatrix} H_{11} - E_p & H_{12} & \cdots & H_{1\mathrm{g}} \\ H_{21} & H_{22} - E_p & \cdots & H_{2\mathrm{g}} \\ \vdots & \vdots & \ddots & \vdots \\ H_{\mathrm{g}1} & H_{\mathrm{g}2} & \cdots & H_{\mathrm{gg}} - E_p \end{vmatrix} = 0 \tag{15}$$

b

The result of the g-order polynomial in E_{p} with g solutions of perturbed energies belongs to each mixed state. Each state has its own coefficients a_{kp}, which must be found in order to acquire the wavefunctions $\Psi_{J,\tau}$, therefore eq.(14) must be solved for each of the g solutions to E_{p} in order to obtain the coefficients a_{kp} and hence the perturbed wave functions $\Psi_{J,\tau}$ [14].

3. Results and Discussion

3.1. Energy Levels

The calculations were conducted by considering the core at ^{132}Sn for all the investigated nuclei, by utilizing the jj56pn model space, by employing the effective interactions j j56pna, j j56pnb, kh5082, cw5082, and khhe.

Fig. 1-4 displays the comparison we had made between our theoretical predictions for the energy level for $^{133;135}$Sn and $^{133;135}$Sb isotopes by utilizing the effective interactions kh5082, jj56pna, khhe, and jj56cdb and compare the predicted results with the measured data.

Fig. 1 shows a comparison of our estimates for the positive and negative parity states of the ^{133}Sn isotope using the effective interactions stated earlier. The ground-state $7/2^-$ is accurately reproduced by three effective interactions.

The three effective interactions agree reasonably well with their corresponding measured data in our theoretical calculations, with the exception of the khhe effective interaction, which failed to determine the ground state of ^{133}Sn. Fig. 2 depicts the predicted excitation energies for the ^{135}Sn isotope positive and negative states. Except for the khhe effective interaction, which failed to determine the ground state of ^{133}Sn which is (7/2-). Many previously unconfirmed experimental data values have been validated by our theoretical calculations. All effective interactions are included in the computations, and the results are reasonable. The low low-lying energy level was calculated using the same effective interactions by assuming the core at 132Sn, as shown in Figs. 3, 4 for the ^{133,135}Sb isotopes, respectively. Effective interactions appropriately reproduce the ground state for their isotopes. Many uncertain energy levels from the values of the measured have been confirmed in our theoretical. The estimates based on three effective interactions agreed rather well with the experimental findings. Except for the khhe effective interaction, which was unable to determine the ground state of ^{133,135}Sb.

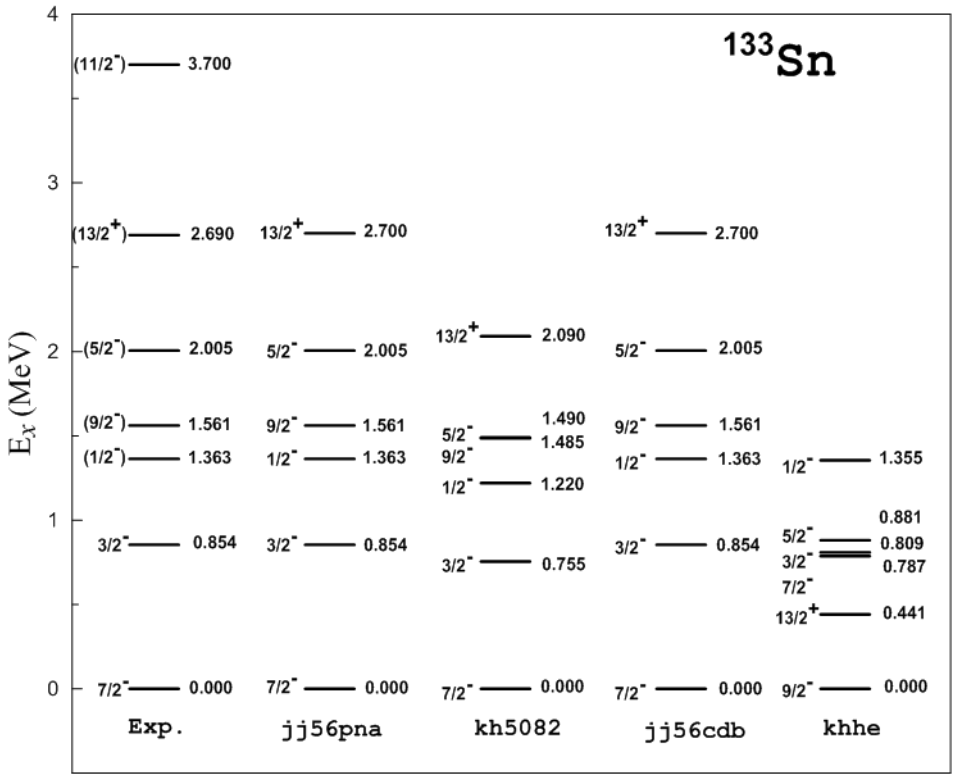

Fig 1. Depicted the comparison of measured and [16] calculated yrast levels for ^{133}Sn isotopes using j j56pna, j j56pnb, kh5082, cw5082, and khhe interactions

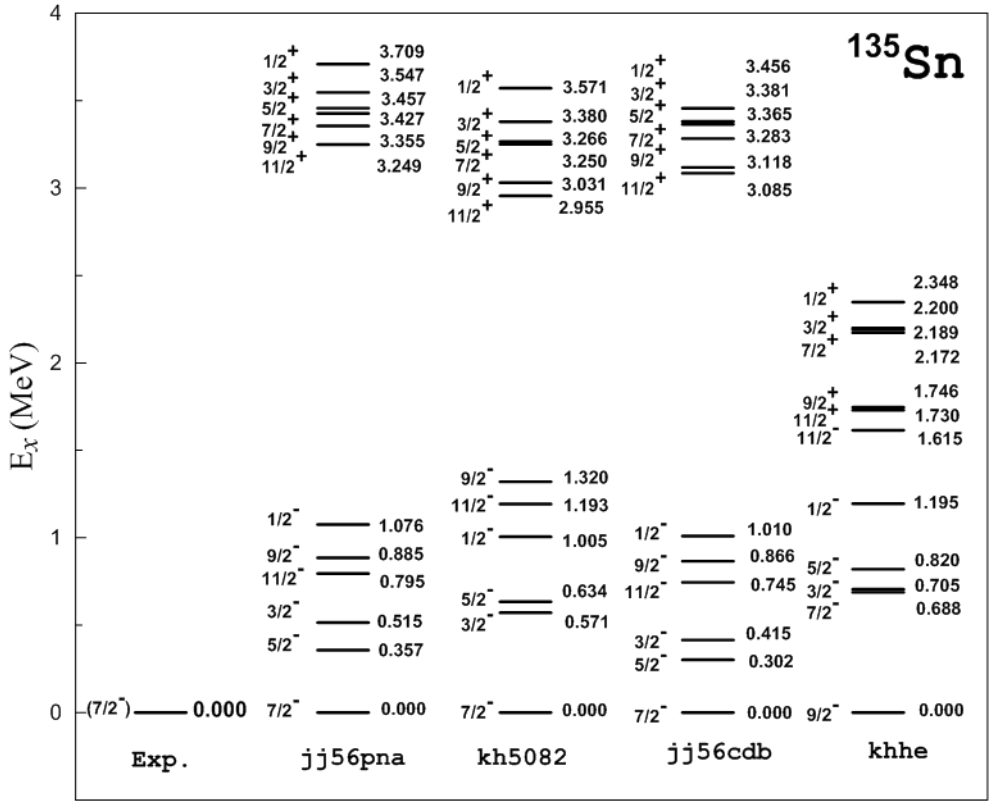

Fig 2. Depicted the comparison of measured [16] and calculated yrast levels for ^{135}Sn isotopes using kh5082, jj56pna, khhe, and jj56cdb interactions

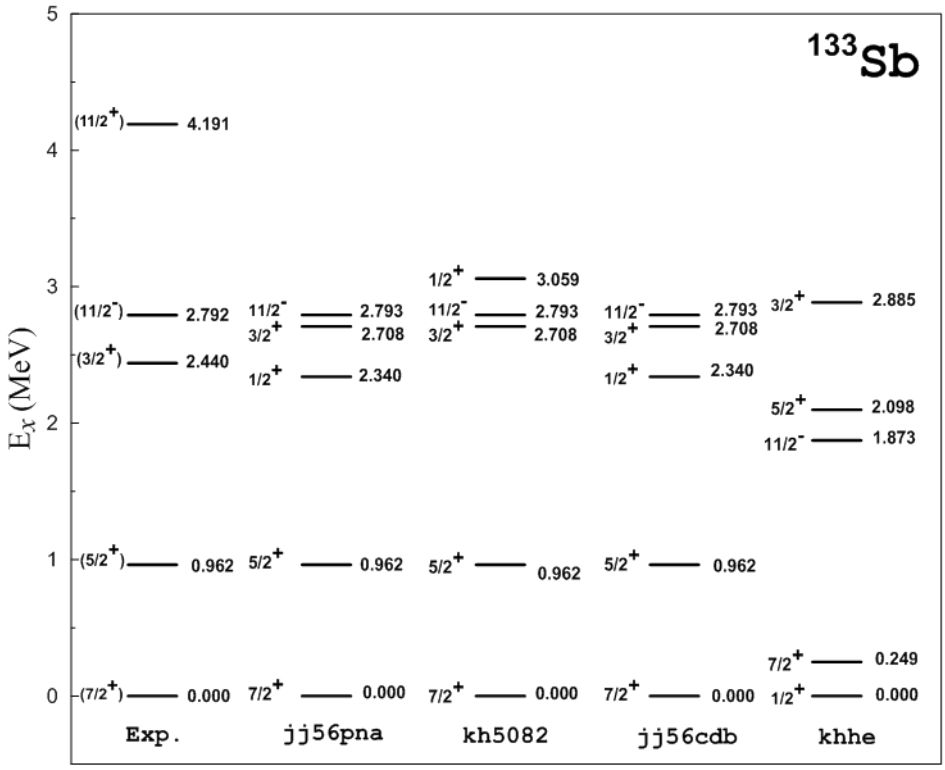

Fig 3. Depicted the comparison of measured [16] and calculated yrast levels for ^{133}Sb isotopes using kh5082, jj56pna, khhe, and jj56cdb interactions

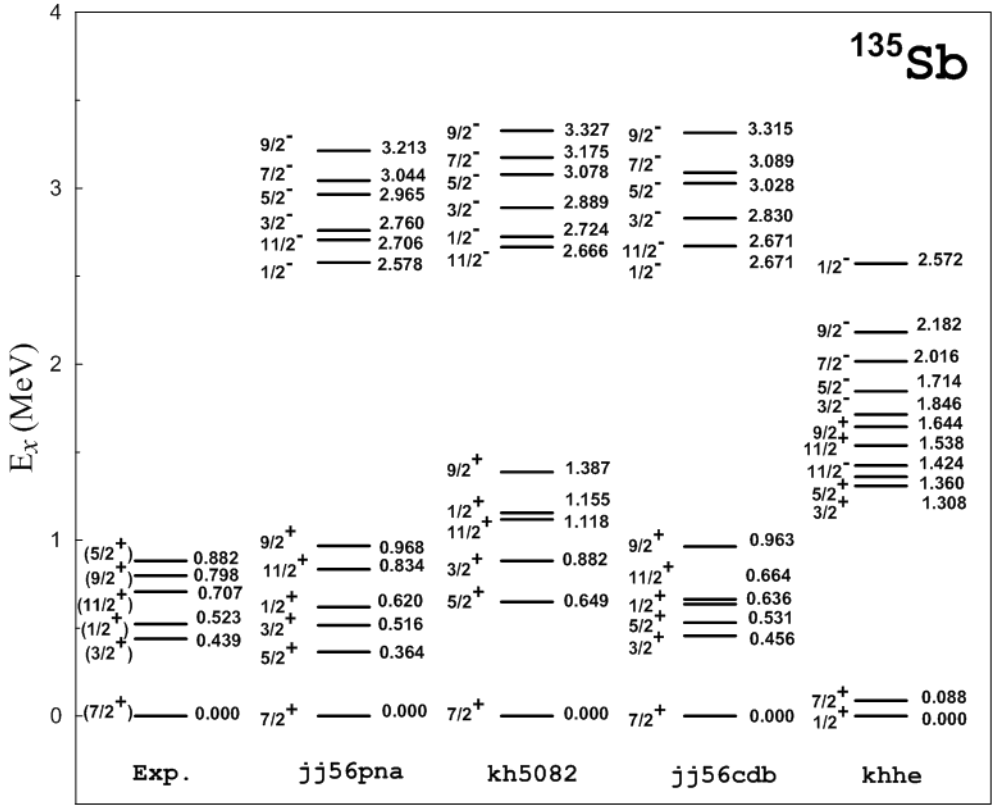

Fig 4. Depicted the comparison of measured [16] and calculated yrast levels for ^{135}Sb isotopes using kh5082, jj56pna, khhe, and jj56cdb interactions

3.2. Energy Levels

Table 1 shows the comparison between the estimated binding energies for all the isotopes investigated in the study by utilizing the four effective interactions indicated previously with their corresponding experimental binding energies [17]. The effective interactions employed in this work can correctly predict the binding energies of all isotopes under examination.

Table 1 compares the estimated binding energy in units of (MeV) for each effective interaction with experimental data from [17]

Isotopes	Exp.	Jj56pna	jj56cdb	Kh5082	Kh5082
^{133}Sn	134.468	151.446	151.129	151.251	153.619
^{135}Sn	168.570	185.228	185.256	185.197	289.119
^{133}Sb	198.257	214.709	214.753	214.723	367.619
^{135}Sb	216.681	233.155	233.117	233.140	315.119

4. Conclusion

Unrestricted large-scale shell model calculations were conducted in this work to investigate the energy levels and binding energies of A-odd nuclei $^{133;135}$Sn and $^{133;135}$Sb using the effective interactions kh5082, jj56pna, khhe, and jj56cdb. The calculated results were compared to the most recent experimental data, and good global agreements were obtained for all interactions except the khhe effective interaction, which failed to determine the ground state of odd nuclei, leading to the conclusion that there is no universal effective interaction in this mass region.

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